Pearson Edexcel Level 3

GCE Mathematics

Advanced

Paper 2: Pure Mathematics

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Time: 2 hours	Paper Reference(s)
	9MA0/02

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. $$y = \sqrt{(2^x + x)}$$

a. Complete the table below, giving the values of y to 3 decimal places.

x	0	0.2	0.4	0.6	0.8	1
y	1	1.161	1.311			1.732

(1)

b. Use the trapezium rule with all the values of y from your table to find an approximation for the value of

$$\int_0^1 \sqrt{(2^x + x)} \, \mathrm{d}x$$

giving your answer to 3 significant figures.

(3)

Using your answer to part (b) and making your method clear, estimate

c. $\int_0^{0.5} \sqrt{(2^{2x} + 2x)} \;\; \mathrm{d}x$

(2)

(Total for Question 1 is 6 marks)

2.

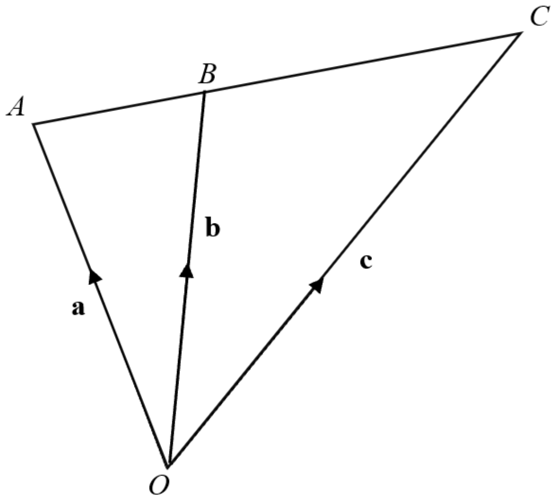

Figure 1

Figure 1 shows a triangle OAC where OB divides AC in the ratio 2 : 3.

Show that $\mathbf{b} = \frac{1}{5}(3\mathbf{a} + 2\mathbf{c})$

(3)

(Total for Question 2 is 3 marks)

3. Use the laws of logarithms to solve the equation

$$2 + \log_2(2x + 1) = 2\log_2(22 - x)$$

(6)

(Total for Question 3 is 6 marks)

4. In the binomial expansion of $(2 - kx)^{10}$ where k is a non-zero positive constant.

The coefficient of x^4 is 256 times the coefficient of x^6.

Find the value of k.

(3)

(Total for Question 4 is 3 marks)

5. a. Given that

$$\frac{x^2 - 1}{x + 3} \equiv x + P + \frac{Q}{x + 3}$$

find the value of the constant P and show that $Q = 8$

(2)

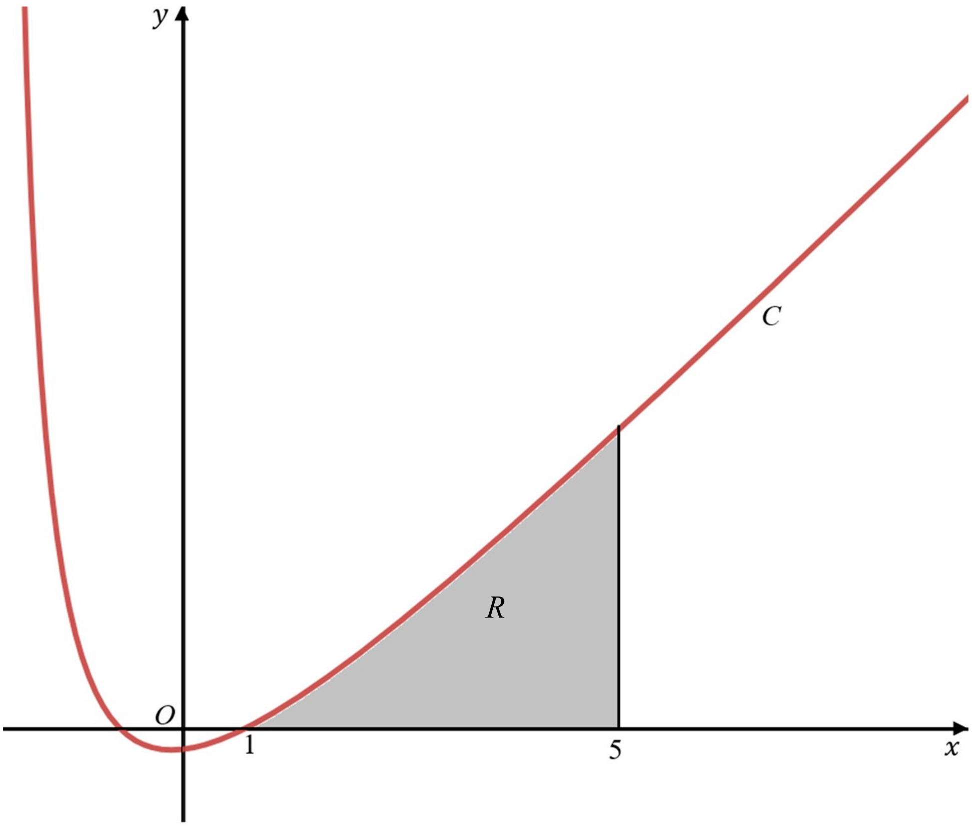

Figure 3

The curve C has equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \frac{x^2-1}{x+3} \qquad x > -3$$

Figure 3 shows a sketch of the curve C.

The region R, shown shaded in Figure 4, is bounded by C, the x-axis and the line with equation $x = 5$.

b. Find the exact area of R, writing your answer in the form $a \ln 2$, where a is constant to be found.

(4)

(Total for Question 5 is 6 marks)

6.

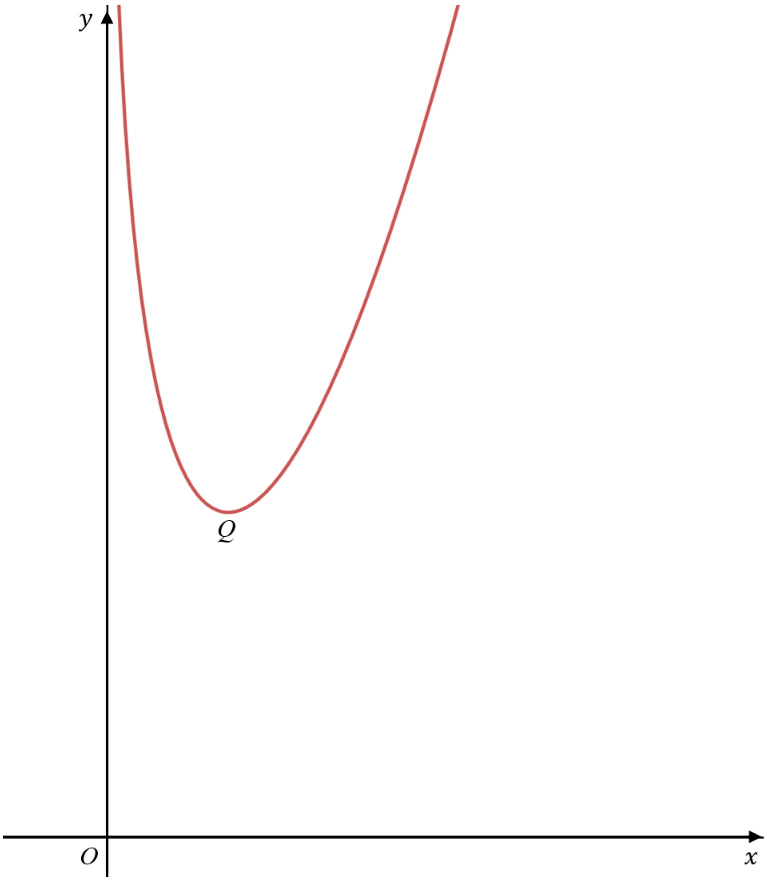

Figure 4

Figure 4 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{2x^2 - x}{\sqrt{x}} - 2\ln\left(\frac{x}{2}\right), \qquad x > 0$$

The curve has a minimum turning point at Q, as shown in Figure 4.

a. Show that $\mathrm{f}'(x) = \frac{6x^2 - x - 4\sqrt{x}}{2x\sqrt{x}}$

(4)

b. Show that the x-coordinate of Q is the solution of

$$x = \sqrt{\frac{x}{6} + \frac{2\sqrt{x}}{3}}$$

(2)

To find an approximation for the x-coordinate of Q, the iteration formula

$$x_{n+1} = \sqrt{\frac{x_n}{6} + \frac{2\sqrt{x_n}}{3}}$$

is used.

c. Taking $x_0 = 0.8$, find the values of x_1, x_2 and x_3.

Give your answers to 3 decimal places.

(3)

(Total for Question 6 is 9 marks)

7. A curve C has equation $y = \mathrm{f}(x)$.

Given that

- $\mathrm{f}'(x) = 18x^2 + 2ax + b$
- the y-intercept of C is -48
- the point A, with coordinates $(-1,45)$ lies on C

a. Show that $a - b = 99$

(4)

The tangent to C at the point A has gradient -84.

b. Find the value of a and the value of b.

(3)

c. Show that $(2x + 1)$ is a factor of $\mathrm{f}(x)$.

(2)

(Total for Question 7 is 9 marks)

8.

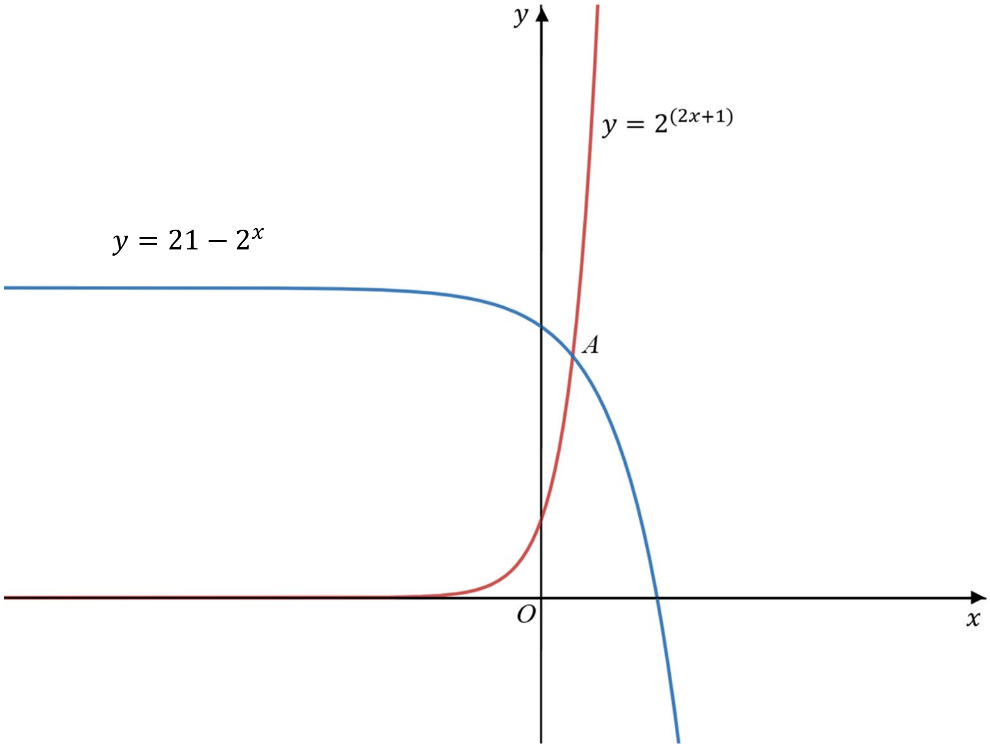

Figure 2

The curves with equation $y = 21 - 2^x$ meet the curve with equation $y = 2^{2x+1}$ at the point A as shown in Figure 2.

Find the exact coordinates of point A.

(4)

(Total for Question 8 is 4 marks)

9. A cup of tea is cooling down in a room.

The temperature of tea, θ°C, at time t minutes after the tea is made, is modelled by the equation

$$\theta = A + 70e^{-0.025t}$$

where A is a positive constant.

Given that the initial temperature of the tea is 85°C

a. find the value of A.

(1)

b. Find the temperature of the tea 20 minutes after it is made.

(2)

c. Find how long it will take the tea to cool down to 43°C.

(4)

(Total for Question 9 is 7 marks)

10. a. Show that

$$\sin 3A \equiv 3\sin A - 4\sin^3 A$$

(4)

b. Hence solve, for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ the equation

$$1 + \sin 3\theta = \cos^2\theta$$

(3)

(Total for Question 10 is 7 marks)

11. a. Sketch the graph of the function with equation

$$y = 11 - 2|2 - x|$$

Stating the coordinates of the maximum point and any points where the graph cuts the y-axis.

(3)

b. Solve the equation

$$4x = 11 - 2|2 - x|$$

(2)

A straight line l has equation $y = kx + 13$, where k is a constant.

Given that l does not meet or intersect $y = 11 - 2|2 - x|$

c. find the range of possible value of k.

(3)

(Total for Question 11 is 8 marks)

12.

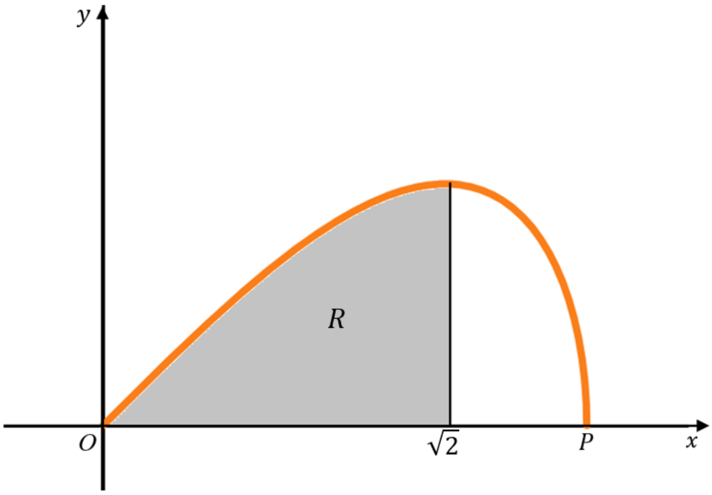

Figure 5

Figure 5 shows part of the curve C with parametric equations

$$x = 2\cos\theta \qquad y = \sin 2\theta \qquad 0 \le \theta \le \frac{\pi}{2}$$

The region R, shown shaded in figure 5, is bounded by the curve C, the line $x = \sqrt{2}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid revolution.

a. Show that the volume of the solid of revolution formed is given by the integral.

$$k\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^3\theta\cos^2\theta \quad d\theta$$

where k is a constant.

(5)

b. Hence, find the exact value for this volume, giving your answer in the form

$p\pi\sqrt{2}$ where p is a constant.

(5)

(Total for Question 12 is 10 marks)

13. The function g is defined by

$$g(x) = \frac{2e^x - 5}{e^x - 4} \qquad x \neq k, \;\; x > 0$$

where k is a constant.

a. Deduce the value of k.

(1)

b. Prove that

$$g'(x) < 0$$

For all values of x in the domain of g.

(3)

c. Find the range of values of a for which

$$g(a) > 0$$

(2)

(Total for Question 13 is 6 marks)

14. A circle C has equation $x^2 + y^2 - 6x - 14y = 40$.

The line l has equation $y = x + k$, where k is a constant.

a. Show that the x-coordinate of the points where C and l intersect are given by the solutions to the equation

$$2x^2 + (2k - 20)x + k^2 - 14k - 40 = 0$$

(2)

b. Hence find the two values of k for which l is a tangent to C.

(4)

(Total for Question 14 is 6 marks)

15. An infinite geometric series has first four terms $1 - 2x + 4x^2 - 8x^3 + \cdots$. The series is convergent.

a. Find the set of possible values of x for which the series converges.

(2)

Given that $\sum_{r=1}^{\infty}(-2x)^{r-1} = 8$,

b. calculate the value of x.

(3)

(Total for Question 15 is 5 marks)

16. Prove by contradiction that if n^2 is a multiple of 3, n is a multiple of 3.

(5)

(Total for Question 16 is 5 marks)